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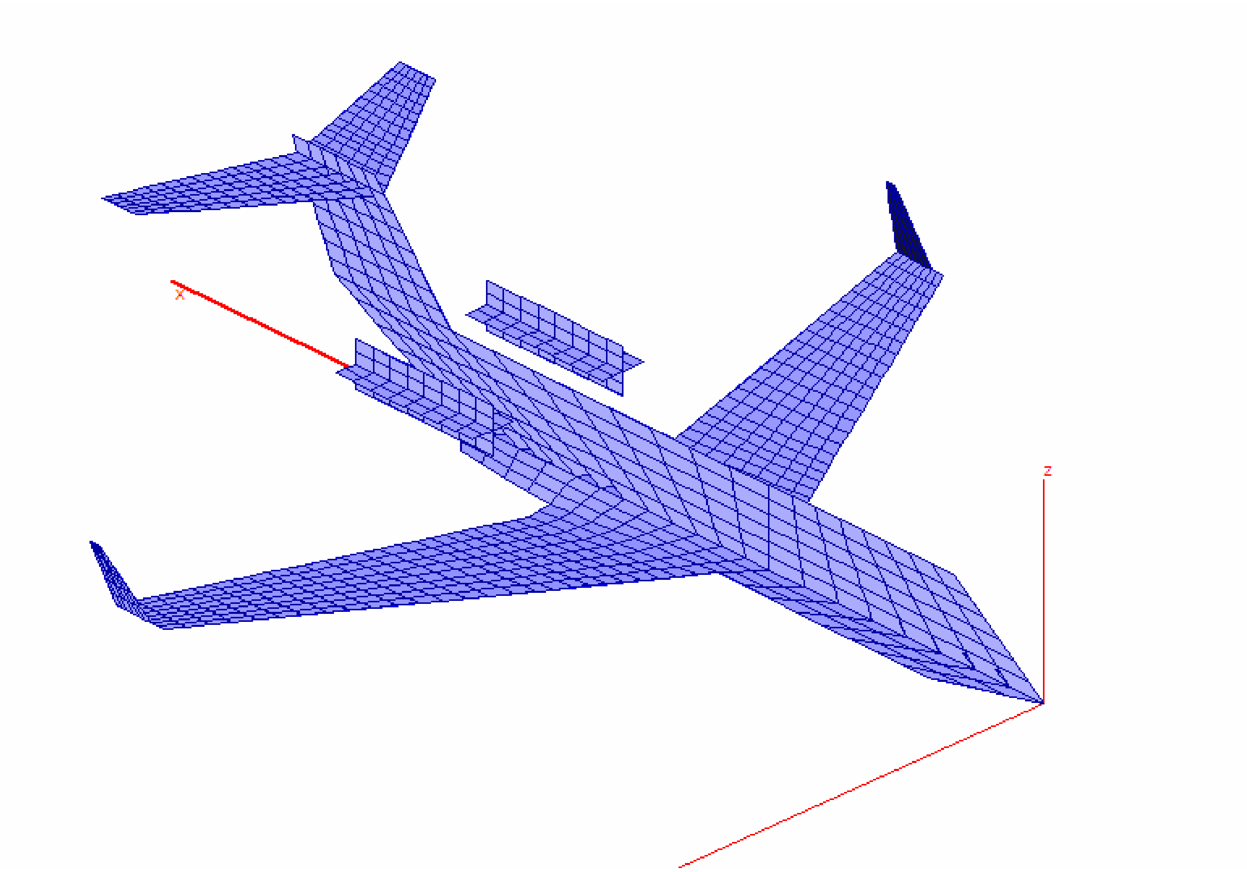
SURFACES

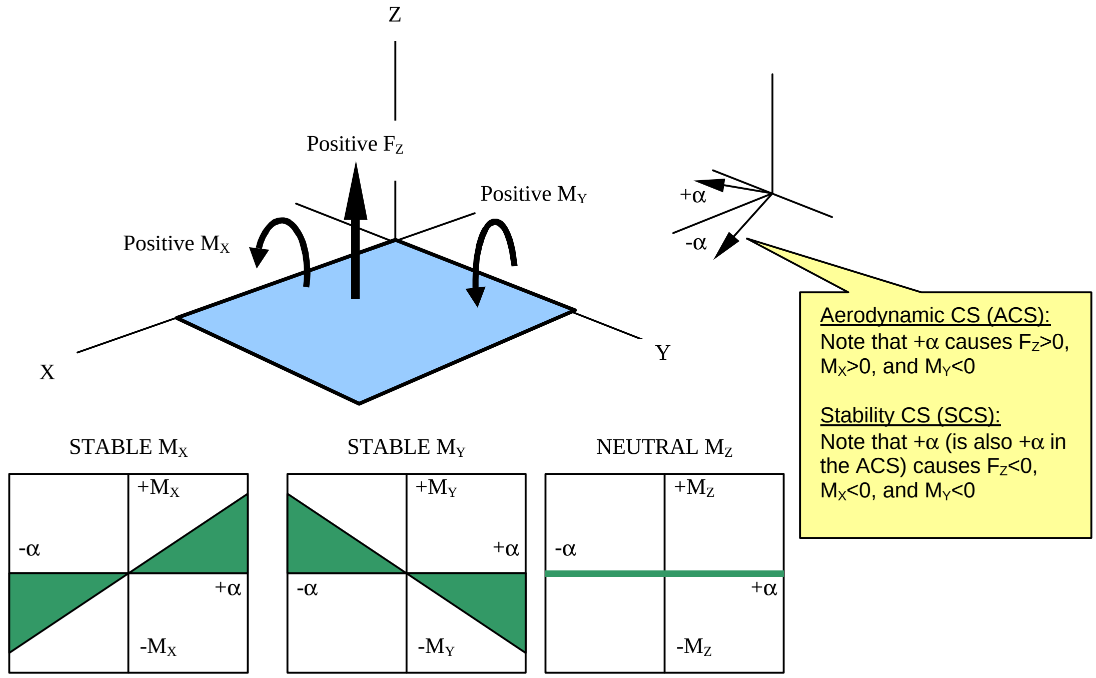
STABILITY ANALYSIS MODULE

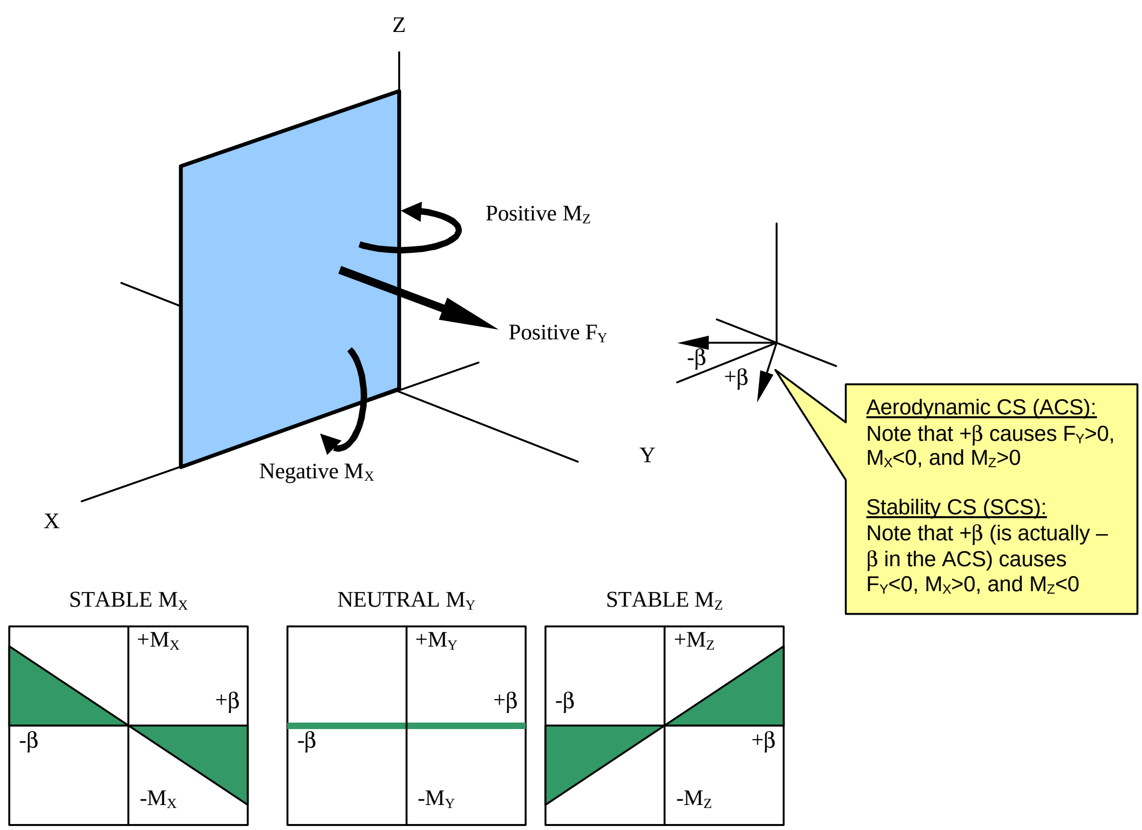


User Manual

SURFACES - Stability Analysis Module

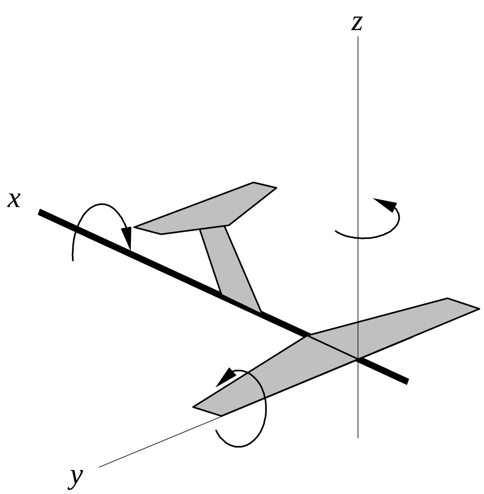
Orientation of Forces and Moments



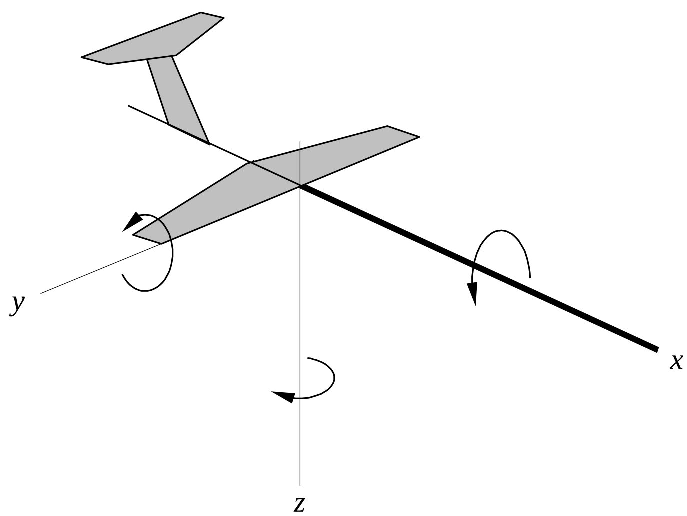


Force and Moment Nomenclature

Name	SURFACES Symbol	Other names
Axial force (along X-axis)	FX	X
Side force (along Y-axis)	FY	Y
Normal force (along Z-axis)	FZ	Z
Rolling moment (about X-axis)	MX	L
Pitching moment (about Y-axis)	MY	M
Yawing moment (about Z-axis)	MZ	N
Coefficient of axial force (along X-axis)	Cx	C_x
Coefficient of side force (along Y-axis)	Cy	C_y
Coefficient of normal force (along Z-axis)	Cz	C_z
Coefficient of rolling moment (about X-axis)	Cl	C_l
Coefficient of pitching moment (about Y-axis)	Cm	C_m
Coefficient of yawing moment (about Z-axis)	Cn	C_n



Standard right-handed Aerodynamic Coordinate System (ACS).



Typical right-handed Stability Coordinate System (SCS).

Note 1: Positive rotation about an axis is always in the direction of the thumb of the right hand, as can be seen in the above figure.

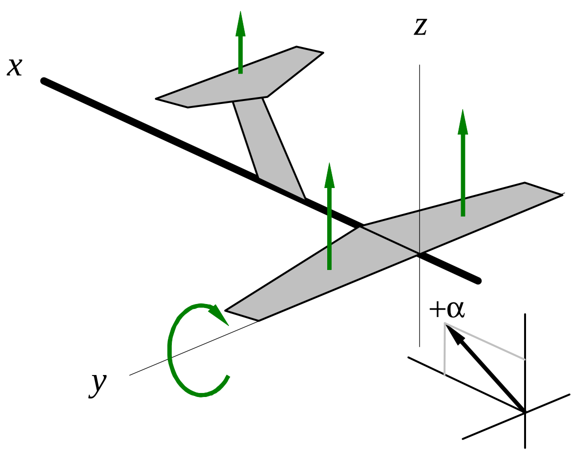
Note 2: **SURFACES** uses a standard right handed Aerodynamic Coordinate System (ACS), which is conventionally used for other aspects of aircraft aerodynamic analyses. In this coordinate system, the sign of the lift is positive, when pointing upwards (i.e. towards positive Z), and the sign of the drag is positive, when pointing backwards (i.e. towards positive X). The user must be cognizant of the orientation of the axes when interpreting results.

Note 3: **SURFACES** comes with a routine that will convert stability derivatives to a standard body axes Stability Coordinate System (SCS). This is typically the default for stability and control related tools.

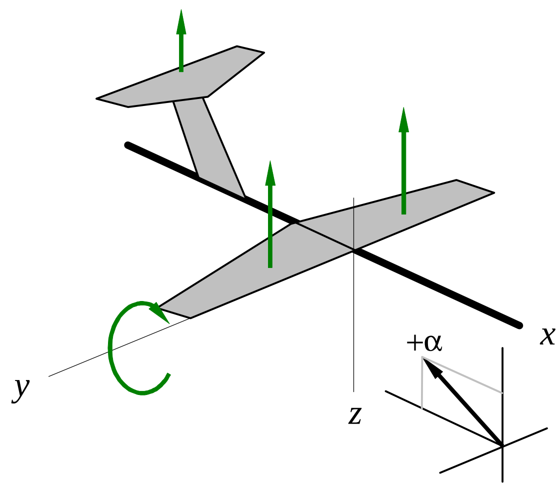
Aerodynamic versus Stability Coordinate System

Increase in Angle of Attack

Standard right-handed Aerodynamic Coordinate System (ACS).



Typical right-handed Stability Coordinate System (SCS).



Increase in a (in Aerodynamic CS)
Values in VLM

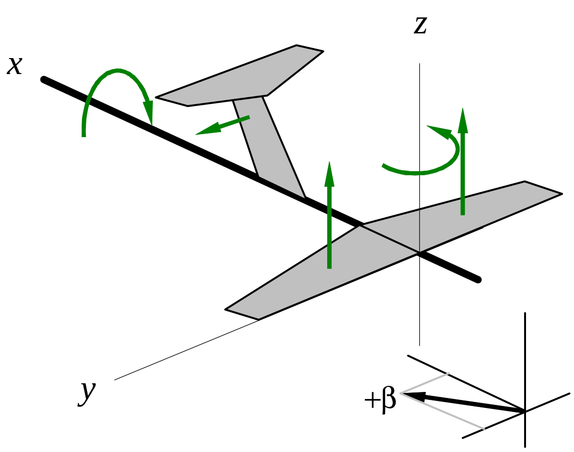
Cx ↑ (drag becomes more positive)	Cxa > 0
Cy = 0 (no side force for a symmetric aircraft)	Cya = 0
Cz ↑ (lift becomes more positive)	Cza > 0
Cl = 0 (no roll for a symmetric aircraft)	Cla = 0
Cm ↓ (nose pitch down more negative)	Cma < 0
Cn = 0 (no yaw for a symmetric aircraft)	Cna = 0
CL ↑	CLa > 0
CD ↑	CDa > 0

Increase in a (in Stability CS)
Values in STAB

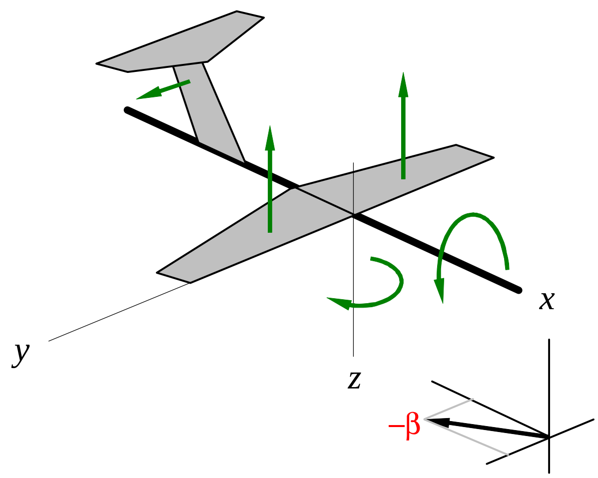
Cx ↓ (drag becomes more negative)	Cxa < 0
Cy = 0 (no side force for a symmetric aircraft)	Cya = 0
Cz ↓ (lift becomes more negative)	Cza < 0
Cl = 0 (no roll for a symmetric aircraft)	Cla = 0
Cm ↓ (nose pitch down more negative)	Cma < 0
Cn = 0 (no yaw for a symmetric aircraft)	Cna = 0
CL ↑	CLa > 0
CD ↑	CDa > 0

Increase in Angle of Yaw

Standard right-handed Aerodynamic Coordinate System (ACS).



Typical right-handed Stability Coordinate System (SCS).



Increase in b (in Aerodynamic CS)
Values in VLM

Cx ↑ or ↓ (expect a small change)	Cxb small
Cy > 0 (if β is positive)	Cyb > 0
Cz ↑ or ↓ (expect a small change)	Czb small
Cl < 0 (for a restoring dihedral effect)	Clb < 0
Cm ↓ (expect a small change)	Cmb small
Cn > 0 (yaw for a symmetric aircraft)	Cnb > 0
CL ↑ or ↓	CLb small
CD ↑ or ↓	CDb small

Increase in b (in Stability CS)
Values in STAB

Cx ↑ or ↓ (expect a small change)	Cxb small
Cy < 0 (if β is positive)	Cyb < 0
Cz ↑ or ↓ (expect a small change)	Czb small
Cl < 0 (for a restoring dihedral effect)	Clb < 0
Cm ↑ or ↓ (expect a small change)	Cmb small
Cn > 0 (yaw for a symmetric aircraft)	Cnb > 0
CL ↑ or ↓	CLb small
CD ↑ or ↓	CDb small

Standard Formulation of Stability Parameters

The following expressions can be used when other method fail to yield results. Note that selected formulation is obtained from Chapter 4 in Reference 1. The user must understand limitations of each for proper use.

General Forces and Moments		
Description	**SURFACES Symbol**[1]	**Formula**
Total force in the X-direction of a plane-fixed global coordinate system.	FX	$F_X = \iint\limits_{Surface} dF_X$
Total force in the Y-direction of a plane-fixed global coordinate system.	FY	$F_Y = \iint\limits_{Surface} dF_Y$
Total force in the Z-direction of a plane-fixed global coordinate system.	FZ	$F_Z = \iint\limits_{Surface} dF_Z$
Total moment about the X-axis of a plane-fixed global coordinate system.	MX	$l = M_X = \iint\limits_{Surface} \left(\vec{r} \times d\vec{F}\right)_X$
Total moment about the Y-axis of a plane-fixed global coordinate system.	MY	$m = M_Y = \iint\limits_{Surface} \left(\vec{r} \times d\vec{F}\right)_Y$
Total moment about the Z-axis of a plane-fixed global coordinate system.	MZ	$n = M_Z = \iint\limits_{Surface} \left(\vec{r} \times d\vec{F}\right)_Z$

Steady State Coefficients		
Description	**SURFACES Symbol**	**Formula**
Drag coefficient	CD	$C_D = C_{D_0} + C_{D_i} = \dfrac{2D}{\rho \cdot V^2 \cdot S}$

[1] The symbols are recommended as some of them are recognized internally by SURFACES.

Induced drag	CDi	$C_{D_i} = \frac{C_L^2}{\pi \cdot AR \cdot e}$
Lift coefficient	CL	$C_L = C_{L_0} + \alpha \cdot C_{L_\alpha} = \frac{2L}{\rho \cdot V^2 \cdot S}$
Rolling moment coefficient	Cl[2]	$C_l = C_{M_X} = \frac{2M_X}{\rho \cdot V^2 \cdot S \cdot b}$
Pitching moment coefficient	Cm[3]	$C_m = C_{M_Y} = \frac{2M_Y}{\rho \cdot V^2 \cdot S \cdot C}$
Yawing moment coefficient	Cn[4]	$C_n = C_{M_Z} = \frac{2M_Z}{\rho \cdot V^2 \cdot S \cdot b}$
Force in the X-direction.	Cx[5]	General form: $C_X = C_T - C_L \cdot \sin\alpha - C_D \cdot \cos\alpha$ Small angle relation: $C_X \approx C_T - C_L\alpha - C_D$
Force in the Y-direction. (Main contributions made are by VT and the fuselage)	Cy[6]	Tail contribution: $C_Y = -\left(C_{L_\alpha}\right)_{VT}(\beta - \sigma)\frac{S_{VT}}{S}$
Force in the Z-direction.	Cz[7]	General form: $C_Z = -\left(C_L \cdot \cos\alpha - C_D \cdot \sin\alpha\right)$ Small angle relation: $C_Z \approx -\left(C_L - C_D \cdot \alpha\right)$
Thrust coefficient.	Ct	$C_T = \frac{2T}{\rho \cdot V^2 \cdot S}$

[2] Note that Cl derivatives are referred to as Cla, Clb, Clta, Clu, Clp, Clq, Clr.
[3] Note that Cm derivatives are referred to as Cma, Cmb, Cmta, Cmu, Cmp, Cmq, Cmr.
[4] Note that Cn derivatives are referred to as Cna, Cnb, Cnta, Cnu, Cnp, Cnq, Cnr.
[5] Note that Cx derivatives are referred to as Cxa, Cxb, Cxta, Cxu, Cxp, Cxq, Cxr.
[6] Note that Cy derivatives are referred to as Cya, Cyb, Cyta, Cyu, Cyp, Cyq, Cyr.
[7] Note that Cz derivatives are referred to as Cza, Czb, Czta, Czu, Czp, Czq, Czr.

Directional stability contribution of vertical tail	Cnvt	$(C_n)_{VT} = -V_V\left(\frac{V_{VT}}{V}\right)^2 (C_L)_{VT}$

AOA Derivatives		
Description	**SURFACES Symbol**	**Formula**
Lift curve slope	CLa	$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha} = \frac{2\pi \cdot AR}{2 + \sqrt{\frac{AR^2\beta^2}{\kappa^2}\left(1 + \frac{\tan^2 \Lambda_{C/2}}{\beta^2}\right) + 4}}$
Drag curve slope	CDa	$C_{D\alpha} = \frac{\partial C_D}{\partial \alpha} = \frac{2C_L}{\pi \cdot AR \cdot e} C_{L_\alpha}$
FX variation with AOA.	Cxa	General form: $C_{X_\alpha} = \frac{\partial C_X}{\partial \alpha} = \frac{\partial C_T}{\partial \alpha} + C_L + \alpha \frac{\partial C_L}{\partial \alpha} - \frac{\partial C_D}{\partial \alpha}$ Since C_T is typically not dependent on α, this becomes: $C_{X_\alpha} = C_L - \frac{\partial C_D}{\partial \alpha} = C_L - \frac{2C_L}{\pi \cdot AR \cdot e} C_{L_\alpha}$ as shown in Reference 1. Note that the subscript 0 in the reference document is omitted here, but this refers to a reference condition. SURFACES always uses CL and CD where Reference 1 uses C_{Lo} and C_{Do}.
FY variation with AOA.	Cya	Should be zero for a symmetric airplane. $C_{Y_\alpha} = \frac{\partial C_Y}{\partial \alpha}$
FZ variation with AOA.	Cza	$C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha} = -\left(C_{L_\alpha} + C_D + \alpha \frac{\partial C_D}{\partial \alpha}\right) = -\left(C_{L_\alpha} + C_D\right)$
Rolling moment variation with AOA.	Cla	Should be zero for a symmetric airplane: $C_{M_{X_\alpha}} = \frac{\partial C_{M_X}}{\partial \alpha} = \frac{\partial C_l}{\partial \alpha}$

Static stability derivative	`Cma`	$$C_{M_\alpha} = \frac{\partial C_{M_Y}}{\partial \alpha} = C_{L_\alpha}\left(X_{neu} - X_{cg}\right)$$
Yawing moment variation with AOA.	`Cna`	Should be zero for a symmetric airplane. $$C_{M_{Z_\alpha}} = \frac{\partial C_{M_Z}}{\partial \alpha} = \frac{\partial C_n}{\partial \alpha}$$

AOY Derivatives		
Description	**SURFACES Symbol**	**Formula**
Lift curve variation with AOY	`CLb`	
Drag curve variation with AOY	`CDb`	
Drag force derivative	`Cxb`	Should be a symmetric curve, with a zero when AOY=0°.
Side force derivative	`Cyb`	Tail contribution: $\left(C_{Y_\beta}\right)_{VT} = -\left(C_{L_\alpha}\right)_{VT}\left(1 - \frac{\partial \sigma}{\partial \beta}\right)\frac{S_{VT}}{S}$
Lift force derivative	`Czb`	Should be a symmetric curve, with a zero when AOY=0°.
Dihedral effect (rolling moment)	`Clb`	Use USAF DATCOM
Contribution of vertical tail to dihedral effect	`ClbVT`	$$\left(C_{l_\beta}\right)_{VT} = -\left(1 - \frac{\partial \sigma}{\partial \beta}\right)\left(\frac{S_{VT} \cdot z_{VT}}{S \cdot b}\right)\left(\frac{V_{VT}}{V}\right)^2\left(C_{L_\beta}\right)_{VT}$$
Pitching moment variation with AOY.	`Cmb`	Should be a symmetric curve, with a zero when AOY=0°.
Directional stability	`Cnb`	$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$$
Contribution of vertical tail to directional stability	`Cnbvt`	$$\left(C_{n_\beta}\right)_{VT} = -V_V\left(\frac{V_{VT}}{V}\right)^2\left(C_{L_\beta}\right)_{VT}$$

d(AOA)/dt Derivatives		
Description	**SURFACES Symbol**	**Formula**
Rolling moment derivative	CXTA	
Pitching moment derivative	CYTA	
Yawing moment derivative	CZTA	
Drag force derivative	CXTA	
Side force derivative	CYTA	
Lift force derivative	CZTA	

U Derivatives		
Description	**SURFACES Symbol**	**Formula**
Speed damping	Cxu	General form: $$C_{X_u} = V\left(\frac{\partial C_X}{\partial u}\right) = V\left(\frac{\partial C_T}{\partial u} - \frac{\partial C_D}{\partial u}\right) = \frac{2}{\rho \cdot V^2 \cdot S}\frac{\partial T}{\partial u} - 2C_T - M\frac{\partial C_D}{\partial M}$$ Gliders: $$C_{X_u} = -M\frac{\partial C_D}{\partial M}$$ Jets/rockets: $$C_{X_u} = -2C_T - M\frac{\partial C_D}{\partial M} = -2(C_D + C_L \tan\theta) - M\frac{\partial C_D}{\partial M}$$ Constant speed propellers: $$C_{X_u} = -3C_T - M\frac{\partial C_D}{\partial M} = -3(C_D + C_L \tan\theta) - M\frac{\partial C_D}{\partial M}$$
Lift damping	Cyu	Usually zero.
Lift damping	Czu	Variation of vertical force by change in airspeed. By definition this is: $$C_{Z_u} = V\left(\frac{\partial C_Z}{\partial u}\right) = -V\left(\frac{\partial C_L}{\partial u}\right) = -M\left(\frac{\partial C_L}{\partial M}\right)$$ Get by a first or a second order curvefit of **CL versus U** in radians. This tends to be small, except at transonic speeds. Per Reference 1, theoretical values are easily calculated for high AR straight wings from: $$C_{Z_u} = -\frac{M^2}{1-M^2}C_{L_\alpha}$$ The derivative impacts phugoid mode by changing the damping.

Rolling moment derivative	`ClU`	Usually zero.
Pitching moment derivative	`Cmu`	Change in pitching moment with airspeed. May be due to compressibility or aeroelastic reasons, but primarily due to shift in center of pressure at transonic speeds. Theoretically, this is given by: $$C_{m_u} = M\frac{\partial C_{MY}}{\partial M} + \rho \cdot V^2 \frac{\partial C_{MY}}{\partial\left(\frac{1}{2}\rho \cdot V^2\right)}$$ The derivative impacts phugoid mode by changing the frequency of oscillation (i.e. period).
Yawing moment derivative	`Cnu`	Usually zero.

P (Roll Rate) Derivatives		
Description	**SURFACES Symbol**	**Formula**
Drag force derivative	`Cxp`	Negligible.
Damping in roll derivative (mostly a wing contribution)	`Cyp`	
FY variation with P. (Often negligible, but mostly impacted by wing and VT).	`Cypvt`	VT contribution: $$\left(C_{yp}\right)_{VT} = -\left(C_{L_\alpha}\right)_{VT}\frac{S_{VT}}{S}\left(\frac{2z_{VT}}{b} - \frac{\partial\sigma}{\partial\left(\frac{P \cdot b}{2 \cdot V}\right)}\right)$$
Lift force derivative	`Czp`	
Damping in roll derivative	`Clp`	No simple formula.
Pitching moment derivative	`Cmp`	Negligible.
Cross derivative due to roll	`Cnp`	Negligible.
Contribution of the VT to the Cross derivative due to roll	`Cnpvt`	VT contribution: $$\left(C_{np}\right)_{HT} = \left(C_{L_\alpha}\right)_{VT} V_V \left(\frac{2z_{HT}}{b} - \frac{\partial\sigma}{\partial\left(\frac{P \cdot b}{2 \cdot V}\right)}\right)$$

Q (Pitch Rate) Derivatives		
Description	**SURFACES Symbol**	**Formula**
Drag force derivative	`Cxq`	Negligible, unless wing is swept or of low aspect ratio.
Side force derivative	`Cyq`	Negligible, unless wing is swept or of low aspect ratio.
FZ variation with Q.	`Czq`	Negligible, unless wing is swept or of low aspect ratio.
Contribution of the horizontal tail to the FZ variation with Q.	`Czqht`	$$\left(C_{zq}\right)_{HT} = -2\left(C_{L_\alpha}\right)_{HT} V_H$$
Rolling moment derivative	`Clq`	Negligible, unless wing is swept or of low aspect ratio.
MY variation with Q, etc.	`Cmq`	Negligible, unless wing is swept or of low aspect ratio.
Contribution of the horizontal tail to the MY variation with Q.	`Cmqht`	

		$$\left(C_{mq}\right)_{HT} = -2\left(C_{L_\alpha}\right)_{HT} V_H \left(\frac{L_{HT}}{C}\right)$$
Yawing moment derivative	Cnq	Negligible, unless wing is swept or of low aspect ratio.

R (Yaw Rate) Derivatives		
Description	**SURFACES Symbol**	**Formula**
Drag force derivative	Cxr	Negligible.
Damping in roll derivative (mostly a VT contribution)	Cyr	Most important contribution is normally from the VT.
Vertical tail contribution.	Cyrvt	VT contribution: $$\left(C_{yr}\right)_{VT} = \left(C_{L_\alpha}\right)_{VT} \frac{S_{VT}}{S}\left(\frac{2L_{VT}}{b} + \frac{\partial\sigma}{\partial\left(\frac{P\cdot b}{2\cdot V}\right)}\right)$$
Lift force derivative	Czr	Negligible.
Cross derivative due to yaw	Clr	No simple formula. Caused by increase in lift on one wing and decrease in the other. Additionally, large VT may contribute. Largest at low speeds.
Contribution of the vertical tail to the Cross derivative due to yaw	Clrvt	VT contribution: $$\left(C_{lr}\right)_{VT} = \left(C_{L_\alpha}\right)_{VT} \frac{S_{VT}}{S}\frac{2z_{VT}}{b}\left(2 + \frac{L_{VT}}{b} + \frac{\partial\sigma}{\partial\left(\frac{P\cdot b}{2\cdot V}\right)}\right)$$
Pitching moment derivative	Cmr	
Damping-in-yaw derivative	Cnr	No simple formula. Always negative. Fuselage effects are usually negligible. Mostly affected by wing and VT.
Contribution of the vertical tail to the Damping-in-yaw derivative	Cnrvt	VT contribution: $$\left(C_{nr}\right)_{VT} = -\left(C_{L_\alpha}\right)_{VT} V_V \left(2\frac{L_{VT}}{b} + \frac{\partial\sigma}{\partial\left(\frac{P\cdot b}{2\cdot V}\right)}\right)$$

Miscellaneous Coefficients		
Description	**SURFACES Symbol**	**Formula**
Horizontal Tail Volume	Vht	$$V_H = \frac{L_{HT}\cdot S_{HT}}{C\cdot S}$$

Horizontal Tail Volume	Vvt	$V_V = \frac{L_{VT} \cdot S_{VT}}{b \cdot S}$

Stability Derivatives Computed by SURFACES

Description	Symbol
STEADY STATE COEFFICIENTS	
Basic lift coefficient	CLo
Lift curve slope	CLa
Lift coefficient	CL
Basic drag coefficient (user input)	CDo
Skin friction drag coefficient (user input)	CDf
Induced drag coefficient	CDi
Drag coefficient	CD
Drag coefficient slope	CDa
AOA DERIVATIVES	
FX variation with AOA	Cxa
FY variation with AOA	Cya
FZ variation with AOA	Cza
Rolling Moment wrt AOA	Cla
Pitching Moment wrt AOA	Cma
Yawing Moment wrt AOA	Cna
Variation of thrust with AOA	CTa
Location of neutral point as % of MAC	hn
Longitudinal Static Margin	SM
AOY DERIVATIVES	
FX variation with AOY	Cxb
Side force derivative	Cyb
FZ variation with AOY	Czb
Dihedral Effect	Clb
Pitching Moment wrt AOY	Cmb
Directional Stability	Cnb
U DERIVATIVES	
Drag variation with airspeed (Mach Number)	CDM
Thrust variation with airspeed	CTu
Speed Damping	Cxu
Side force damping	Cyu
Lift force damping	Czu
Rolling moment with U	Clu
Pitching moment with U	Cmu
Yawing moment with U	Cnu
P DERIVATIVES(ROLL)	
Lift variation with P	CLp
Drag variation with P	CDp
FX variation with P	Cxp
Side force due to roll derivative	Cyp
FZ variation with P	Czp
Damping-in-Roll derivative	Clp

Pitching moment variation with P	Cmp
Cross derivative due to roll	Cnp
Q DERIVATIVES(PITCH)	
Lift variation with Q	CLq
Drag variation with Q	CDq
FX variation with Q	Cxq
FY variation with Q	Cyq
FZ variation with Q	Czq
Rolling moment with Q	Clq
Pitching moment with Q	Cmq
Yawing moment with Q	Cnq
R DERIVATIVES(YAW)	
Lift variation with R	CLr
Drag variation with R	CDr
FX variation with R	Cxr
FY variation with R	Cyr
FZ variation with R	Czr
Cross derivative due to yaw	Clr
Pitching moment with R	Cmr
Damping-in-Yaw derivative	Cnr
AILERON DEFLECTION DERIVATIVES(ROLL)	
Lift variation with roll	CLda
Drag variation with roll	CDda
FX variation in roll	Cxda
FY variation in roll	Cyda
FZ variation in roll	Czda
MX variation in roll	Clda
MY variation in roll	Cmda
MZ variation in roll	Cnda
ELEVATOR DEFLECTION DERIVATIVES(PITCH)	
Lift variation with pitch	CLde
Drag variation with pitch	CDde
FX variation in pitch	Cxde
FY variation in pitch	Cyde
FZ variation in pitch	Czde
MX variation in pitch	Clde
MY variation in pitch	Cmde
MZ variation in pitch	Cnde
RUDDER DEFLECTION DERIVATIVES(YAW)	
Lift variation with yaw	CLdr
Drag variation with yaw	CDdr
FX variation in yaw	Cxdr
FY variation in yaw	Cydr
FZ variation in yaw	Czdr
MX variation in yaw	Cldr

MY variation in yaw	Cmdr
MZ variation in yaw	Cndr
HIGH LIFT DEFLECTION DERIVATIVES	
Lift variation with flap	CLdf
Drag variation with flap	CDdf
FX variation in flap	Cxdf
FY variation in flap	Cydf
FZ variation in flap	Czdf
MX variation in flap	Cldf
MY variation in flap	Cmdf
MZ variation in flap	Cndf